APPRAISAL OF ELECTRICAL HEATING SIMULATION RESULTS BY DIFFERENT MODELING TOOLS

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ABSTRACT

Electric resistive heating as a method of heavy and extra-heavy oil (EHO) recovery, has been studied extensively in the last decades and field-tested recently. It is viewed now as a feasible alternative for EHO recovery when surface mining or SAGD are not possible. Numerical simulation of the process at reservoir conditions is one of the key steps in a recovery method design and evaluation. In case of electric heating, however, the simulations are not always straightforward and require special efforts for rigorous model development. The objective of this work is to build an adequate field-scale model of electrically assisted EHO recovery. For instance, one of critical points was to check the validity of electrical three-phase current and power computations. To do this the comparison has been done between different numerical simulators: (1) COMSOL Multiphysics, the finite element simulator based on general approach to solution for partial differential equations, (2) multipurpose numerical simulator TETRAD and (3) the thermal reservoir simulator STARS by CMG which is the target modeling environment. The multiphysics simulator was used as a reference for the stationary (instantaneous) electric power calculations, while the thermal multiphase flow simulator performed robust and accurate transient calculations of electrical current distribution from electrodes.

Series of particular test problems of different size with different properties distribution have been developed including those describing typical Athabasca heterogeneous reservoir conditions adapted for the study purpose. The use of special grid refinement around electrical well improved the calculations in STARS but not in a trivial way. Advantageous grid refinement configurations, possible improvements of computational performance and results, modeling procedure for low-frequency heating assisted EHO recovery and particular case simulation results, are presented and discussed in this work.

INTRODUCTION

The main idea of the low-frequency electric field implementation inside the reservoir is to provide a heating source and to facilitate the oil flow to production wells due to reduction of oil viscosity induced by locally increased temperature. Since the 1970s the low-frequency heating (LFH) method has been developing for about forty years and has already been field-tested (*e.g.* McGee 2008). From the physical viewpoint the method is based on the Joule effect, the original (connate) reservoir water playing a role of conductor. In its current state the method suggests to supply electrical current via the electrodes settled directly in the special wells (E-wells). Thus the heat is generated over the reservoir volume according to electrical current density field. As water saturation, salinity and reservoir temperature affect the bulk medium electrical conductivity, an understanding of governing physical mechanisms of this oil recovery method requires consideration of multicomponent heat and mass transfer under the LFH conditions.

The advantages of the method become evident when the steam injection is not efficient or forbidden like for instance in cases of a shallow or too deep deposit (risk of steam leakage or prohibitive heat loss and/or pressure limitations, respectively), high initial oil viscosity (fluid injectivity and interwell connectivity problems), GHG emission limits etc. The LFH may also be used in combination with other methods to improve ultimate oil recovery (Harvey et al. 1979, Hiebert et al. 1989, Tran et al. 2009). From the other side the typical LFH problems can be related to the control of electrode temperature (to avoid the local water evaporation) and of electric current distribution (e.g. to prevent the energy loss to neighboring aquifer).

Most frequently the development of laboratory and field tests design and their results analysis are performed with the use of dedicated numerical simulators or in-house codes (cf. Hiebert et al. 1986, Killough and Gonzalez 1986, Pizzaro and Trevisan 1990, McGee and Vermeulen 2007; short summary of the resistive heating simulations can be found in Table 1). In particular, the CMG-STARS reservoir simulator offers a dedicated module to perform the electric field computations (e.g. Bogdanov et al. 2010). Making use of the extensive list of keywords it is possible to dynamically control the most important parameters of the LFH: electric potential and/or current per electrode, total heating power, maximum reservoir temperature and others. At the same time note that the standard technical mean to adapt a model to realistic electrode design seems obsolete and additional efforts are required to find out acceptable solution.

The objective of our current work is to build an adequate field-scale model of electrically assisted EHO recovery. Within this scope, one of the critical points is to check the validity of electrical three-phase current and power computations. To do this the comparison has been done between different numerical simulators. The COMSOL Multiphysics, finite element (FE) simulator based on a general approach to solution for partial differential equations, was used as a reference for the stationary (instantaneous) electric power calculations. The multipurpose numerical simulator TETRAD performed the robust and accurate transient calculations of electrical current distribution between the electrodes together with thermal multiphase flow. Finally, the thermal reservoir simulator STARS by CMG was the target modeling environment.

ELECTRIC FIELD MODEL

As it has been stated above, the LFH method is based on well-known Joule effect occurring in the original (connate) reservoir water playing a role of conductor. The electrical current is supplied via electrodes installed inside the E-wells. An industrial three-phase power supply is used for the heating purpose. The governing and constitutive equations for complex electric potential (Appendix A) are obtained in low frequency (or in other word, infinite wave-length) limit from the system of standard electromagnetic equations and Archie's law (Archie 1942). Let's consider the key parameters of the electric field model.

The bulk reservoir electrical conductivity plays an important role in heat generation at LFH. It has recently been concluded that the pre-heating period (initial reservoir heating frequently without or with insignificant production) is technically difficult and may be relatively long especially at low initial reservoir conductivity. At similar applied potential input, a reservoir with higher electrical conductivity generates more electric power which facilitates the pre-heating and can diminish drastically its time. Another physical factor related to the medium electrical conductivity is the effective electrode radius which can be modified (preferably, increased) via forced formation of highly conductive layer around electrode (cf. Yung et al 2003). This can be done, for example, via arranging the circulation of brine from the electrode which increases progressively the brine-saturated zone and hence, the effective electrode size (cf. Bogdanov et al., 2010). The initial reservoir conductivity data used in referenced numerical simulations can be found in Table 1.

It is not so difficult to conclude that among the factors influencing the conductivity according to Archie's law (cf. Eq. A-2, Appendix A) only water saturation and solute concentration variations may provide an order of magnitude increase of initial reservoir conductivity. The mechanisms of such a variation are convective, diffusive and dispersive transport of brine. The convection (e.g. via the water circulation) will dominates near the electrodes and may also become important after opening the production wells at elevated reservoir temperature. Taken in whole the diffusion and especially the mechanical dispersion may become important mechanisms of the solute propagation (or more precisely, the propagation of an elevated conductivity zone). The dispersion coefficient is proportional to local fluid velocity magnitude so that this contribution to the conductivity modification can be controlled. To the best of our knowledge, the idea of the artificial variation of medium conductivity via brine injection was first considered by El-Feky (1977) in the selective resistive heating framework.

The geometry of reservoir, E-wells pattern and electric streamline configuration are also important features of the process. For instance, the choice of the well spacing (L) is not always trivial while the total power (and local power density) scales as L^{-2}. Different values of L found in literature are presented in Table 1. The reservoir heterogeneity (at least, clay and water saturated layers) should always be taken into account and numerical simulations can contribute much to the relevant choice of the LFH design features.

NUMERICAL METHODOLOGY

Numerical simulation of the process at reservoir conditions is one of the key steps in a recovery method design and evaluation. However in case of electric heating the simulations are not always straightforward and require special efforts for rigorous model development. One of the main reasons is the nearly singular electric field in the electrode vicinity. This stems directly from the smallness of electrode radius compared to typical well spacing which is one of the most important physical parameters of the process. Such a singularity requires frequently an adequate grid refinement around E-wells which may be expensive for field-scale modeling. For example, after numerous tests Bogdanov et al. (2010) chose a fine enough grid for the LFH modeling which put some limitations for the full field scale model application.

Another important factor in the simulations is the electrical conductivity and its variation with water salinity, saturation and reservoir temperature according to generalized Archie's law (Eq. A-2). The great conductivity contrast may lead to difficulties in convergence of numerical solution and limit the grid size. Mention also the very possible (and sometimes, desirable, McGee, 2010) temperature conditions favorable for the local water evaporation (not in the electrode vicinity, of course). Rigorous modeling of this situation is only possible if numerical model offers a rigorous coupling between multiphase thermal flow and electric power computations.

To provide a relevant model development and evaluation three different simulators have been used in the current work. The 3D LFH model was developed in CMG-STARS, reference reservoir simulator for thermal methods of oil recovery possessing a special module for the LFH applications. It has been tested using COMSOL and TETRAD simulators with the series of particular specially designed test problems of different size with different properties distribution including those describing typical Athabasca heterogeneous reservoir conditions adapted for the study purposes. The geometry of test cases has been chosen close to the symmetry element of so-called ET-DSP process (cf. McGee, 2008).

The advantage of COMSOL is the great flexibility of the model geometry construction and also of the grid generation procedure including structured or unstructured automatically or manually refined grid near geometrical peculiarities of the model, which makes it an excellent numerical tool to solve the 3D instantaneous electric field problems. One example of unstructured grid generated for the test problem 2 (Figure 1b) is presented in Figure 2. While in practice there are some limitations related mainly to memory requirements, the use of dedicated parallel solvers and again the grid flexibility can help to control the computational performance. Note finally that the COMSOL model can be successfully used for arbitrary distribution of reservoir electric properties and has been chosen recently for the loose coupling with CMG-STARS (Torres et al. 2010) within the similar problem framework. For each test problem upon a series of computations with different grid and other numerical parameters, COMSOL provided the reference instantaneous electric field solution.

The multipurpose simulator TETRAD (Vinsome and Shook 1993) offers a rigorous model of electric current distribution from an E-well which is based on electric-well index definition. The electric-well index definition is similar to standard Peaceman's well index formulation (see Appendix B). As it is shown below this improves the modeling results, in particular, in the vicinity of E-well which makes more reliable the computations of principal electric model parameters such as electric resistance, total power supply etc.

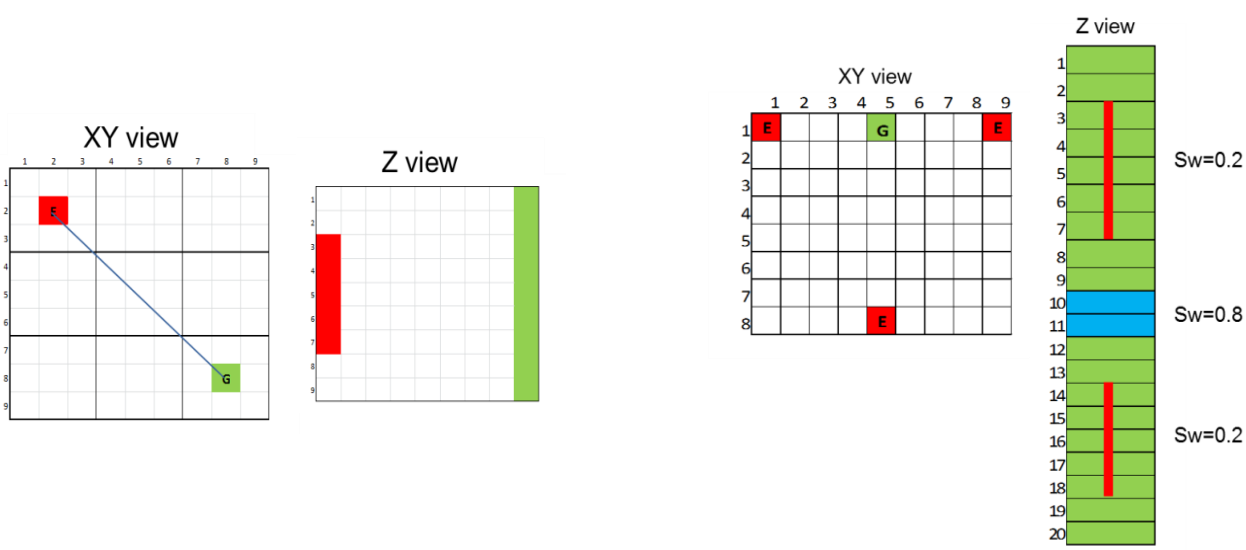
The objective of this work was to build an adequate field-scale model of electrically assisted EHO recovery. The comparison between three simulators resulted in definition of necessary numerical models, determination of their technical parameters and necessary problem-dependent grid for successful field-scale simulations. In the next section we present concisely the main steps and results of this study.

TEST PROBLEM FORMULATIONS AND MAIN RESULTS

To check the validity of electrical current and power computations two study tests were first proposed without fluid transport consideration. The results by three simulators and their comparison underlay the specification of adequate CMG-STARS model illustrated below using the third test problem formulation which included preheating and

Table 1. Typical parameters of simulations cases reported in literature.

Reference	Initial conductivity, [S m^{-1}]	Problem formulation features and parameters
Todd & Howell (1978)	0.005 – 0.043	Radial geometry, homogeneous reservoir, L= 30m, preheating time 700 days.
Harvey et al (1979)	2.5	Case 1: 5 spot pattern, homogeneous reservoir, L=137.2m, preheating time 42 days. Case 2: 5 spot pattern, 2 layers reservoir, L=152.4m, preheating time 28 days.
Hiebert et al (1989)	0.014 – 0.020	Case 1: Homogeneous reservoir, L=40m, preheating time 180 days. Case 2: 2 oil sand + 2 shale layers, L=64m preheating time 547 days. Case 3: Five spot pattern, L=53m 3 oil sand + 2 shale layers, preheating time 365 days.
Pizarro & Trevisan (1990)	0.0004	5 spot pattern, homogeneous reservoir, L=116.4m, preheating time 80 days.
McGee & Vermeulen (2007)	0.1	Element of ET-DSP pattern, homogeneous reservoir, L=16m, preheating time 30 days.
McGee (2008)	0.025 – 0.0063	Pilot ET-DSP pattern, homogeneous reservoir, L=8m, preheating time 36 days.



(a) **(b)**

Figure 1. Electrode configurations for problems 1 (a) and 2 (b): the E-wells (E) and the ground potential well (G).

production of initially immobile bitumen.

PROBLEM 1. This study case represents a simplest (single-phase) electrical heating 3D configuration. The model was built with the total volume of 729 m^3 (9m by 9m by 9m); two electrodes were placed in opposite corners, one at a given voltage (Table 2) and the other at ground (zero) potential. This created the electric current through conductive reservoir medium and the reservoir temperature increased progressively. Figure 1a shows the electrode configuration in the XY plane and electrodes completion in the Z plane; note that the E-well is only active between layers 3 to 7 while ground electrode penetrates the whole reservoir thickness. No heat loss conditions were imposed. The electric field parameters and the main physical properties of the reservoir and its fluids are presented in the first column of Table 2 below.

Table 2. Main model parameters for test problems.

Test problem No	1	2
Reservoir area, m^2	81	72
Reservoir thickness ,m	9	20
Electrode length, m	5	6
Electrode radius, m	0.09	0.09
Electrode potential, V	150 & 480	150
Initial reservoir pressure, bar	10	10
Initial reservoir temperature, °C	11	11
Initial oil saturation	0.8	0.8/0.2
Initial water saturation	0.2	0.2/0.8
Initial oil viscosity, Pa*s	1700	1700
Initial bulk conductivity, S/m	0.005	0.001
Water electrical conductivity, S/m	0.4	0.4

Three grid configurations were tested in electric field calculations using TETRAD and STARS. The *fine* grid model consists in 6561 cells (27 by 27 by 9) with a cell size of 0.33m in I and J directions (ΔX=ΔY) while a cell thickness of 1m (ΔZ) was used for all models. The *medium* grid model consists in 729 cells (9 by 9 by 9) with a cell size of ΔX=ΔY=1m. Finally the *coarse* grid model consists in 81 cells (3 by 3 by 9) with a cell size of ΔX=ΔY=3m. Figure 1a presents the medium grid for STARS and TETRAD and Figure 2 the COMSOL finite-element grid using surface (2a) and volume (2b, example of wireframe visualization for temperature field) views.

As it can be seen in Table 3, in this case study two scenarios were tested at different potential imposed at the E-well (150V and 480V) and simulations were performed in COMSOL, TETRAD and STARS. The electric power calculations for the three grid configurations made with the three simulators are presented in Table 3.

TETRAD simulations were successfully performed for coarse and medium grids. However the fine grid simulations were not possible due to intrinsic cell size restriction. Total electric power calculations were consistent for the different grid size evaluated and deviation from the reference (COMSOL) values was around 4% (cf. Table 3, TETRAD results).

CMG-STARS simulations were performed for all grid options and results were highly dependent on grid size for both voltage scenarios as presented in Table 3. Deviations from the reference (COMSOL) values were between 23% and 41%. It turned out (after examination and analysis of the results) that standard electric filed computation model can't be properly used for the case of the E-well because of limited option for the geometry definition of E-well boundary surface. To improve electric field calculations in CMG-STARS, numerous different configurations of the local grid refinement around electrical wells have been tested and compared to the COMSOL reference model.

The hybrid grid refinement option in STARS, presumes the local radial refinement inside the E-well 3D grid block. Following this approach, the grid block is divided in two regions: the well and the reservoir. In the well region, the fluid flow equations are formulated in cylindrical (or elliptical) coordinates for isotropic (or anisotropic) properties. In the reservoir region, linear streamlines are considered and the fluid flow equation is discretized using Cartesian coordinates. Hybrid refinement results in a local cylindrical grid with axial direction which may be oriented in any of the global directions. A maximum of 10 divisions in the radial direction can be specified per grid block. The angular direction can be divided into either 1 or 4 sectors and the innermost radial division is always a full circle. The well radius (r_w) is required for the hybrid refinement definition in STARS, and represents the inner radius of the innermost ring. Selection of the hybrid grid parameters was evaluated in this study in order to arrive at standard electric-well index definition like those used in TETRAD (cf. Appendix B) and improve electric field calculations around the electrode.

The electric power estimations for the so-defined hybrid refinement case are presented in Table 3 (STARS Hybrid cases). The use of an optimal hybrid refinement around electrodes (E-wells and ground potential wells) improved electrical calculation performance in STARS by decreasing deviation from COMSOL's values below 6% for all studied scenarios.

PROBLEM 2. This study case represents the resistive heating by industrial 3-phase electric current for a half-element of the typical ET-DSP configuration. A 3D model for a reservoir of 1440 m^3 total volume was evaluated (9m by 8m by 20m) as shown in Figure 1b. Grid cell size was one meter at each direction. Three electric wells (E-well) and one ground well were placed to evaluate half ET-DSP configuration. Each E-well consists of two electrodes (6 meters long) placed vertically as illustrated in the Z view of Figure 1b. The peculiarity of this study case is the water saturated layer between two electrodes placed in each E-well. Main physical properties of the reservoir model and fluids are presented in column 2 of the Table 2 above. One example of the simulation results by different tools are presented in Table 4 below. Deviations from COMSOL's value were 2% and 32% for TETRAD and standard STARS models, respectively. The use of hybrid grid refinement in STARS (cf. STARS Hybrid in Table 4) successfully improved the electric power calculation.

Table 3. Electrical heating results for test problem 1.

Simulator Voltage	Total power (W)	
	150V	480V
COMSOL	292.5	2995.7
TETRAD:		
Coarse	279.71	2864.3
Medium	280.75	2874.7
STARS:		
Coarse	225.5	2308.99
Medium	195.6	2001.7
Fine	171.3	-
STARS-Hybrid:		
Coarse	293.71	3007.89
Medium	275.94	2825.59

PROBLEM 3. The third study case represents an isolated 3-phase electrical heating pattern of 4 electrodes (E01 to E04) and a production well as shown in Figure 3. A reservoir of 39,520 m^3 total volume was modeled (26m by 38m by 40m). The E-wells separation makes 16 m, three electrodes are placed vertically in each of them (Figure 3, Z view). No heat loss was considered in the model. Figure 3 also presents the vertical variability of reservoir properties in the model. Initial bulk reservoir conductivity was approximately 0.001 S/m according to Archie's law (Eq. A-2); the temperature dependence of water electrical conductivity was included (cf. McGee and Vermeulen 2007, Appendix A). To control the electrodes temperature the moderate water injection rate (1 m^3/day per well at 40°C) was imposed. Total heating power has been limited to 150 kW.

Table 4. Electrical heating results for test problem 2.

Simulator	Total power (W)
COMSOL	2560.0
TETRAD	2507.1
STARS Standard	1751.7
STARS Hybrid	2561.4

The evolution of reservoir temperature presented in Figure 4 demonstrates the efficient temperature control on each electrode, and fairly homogeneous reservoir temperature field inside the pattern after preheating which results in the significant reduction of initial EHO viscosity. The oil viscosity corresponding to the temperature shown in Figure 4c was within 20-200 mPa.s (after 12 months of heating); the average temperature rise was 26°C, 36°C and 55°C after 4, 7, 12 months, resp. The estimation of the equivalent CPU time indicates that this test problem computation in CMG-STARS was faster than for the 6 times smaller volume problem described in Bogdanov et al. (2010).

CONCLUSIONS

Three simulators have been used to check the validity of electrical three-phase current and power computations. This resulted in adequate field-scale model development using dedicated reservoir simulator.

The test results analysis demonstrated that the E-well index definition is crucial for modeling the resistive heating assisted EHO recovery, at least, in its standard design framework with electrodes installed inside special E-wells. Models of this type become more reliable, less sensitive to the grid choice and the reservoir properties dynamic distributions and thus computationally efficient.

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NOMENCLATURE

EWI	=	electric-well index
f	=	bulk el. conductivity temperature factor
I_{iw}	=	electric current in i-th grid block
J	=	heating power density
m	=	lithology constant (see Archie's law definition, Eq. A-2)
n	=	Archie's law power factor
r_w	=	well radius
R_i	=	i-th grid block resistance factor
S_w	=	water saturation
V_i	=	i-th grid block el. potential
V_w	=	given E-well el. potential
ΔX	=	grid block size in I-direction
ΔY	=	grid block size in J-direction
ΔZ	=	grid block size in K-direction
ε	=	porosity
σ	=	bulk reservoir electric conductivity
σ_i	=	i-th grid block el. conductivity
σ_w	=	connate water electric conductivity
υ	=	complex electric potential

REFERENCES

Archie, G.E. (1942). The Electrical Resistivity Log as an Aid in Determining Some Reservoir Characteristics. *Petroleum Transactions of AIME*, **146**, 54-62.

Bogdanov, I.I., Torres, J.A., Akhlaghi, H.A., Kamp, A.M. (2010). The Influence of Salt Concentration in Injected Water on Low-Frequency Electrical-Heating-Assisted Bitumen Recovery. *SPE Journal*, 16(3), 548-558.

El-Feky, S.A. (1977). *Theoretical and Experimental Investigation of Oil Recovery by the Electrothermic Technique*. (PhD Dissertation), University of Missouri-Rolla, MO.

Harvey, A. H., Arnold, M. D., El-Feky, S. A. (1979). Selective Electric Reservoir Heating. *J. Canadian Petroleum Technology*, 18(3), 47-57.

A. D. Hiebert, F. E. Vermeulen, F. S. Chute, C. E. Capjack (1986). Numerical simulation results for the electrical heating of Athabasca oil-sand formations. *SPE Resevoir Engineering*, **1**(1), 76-84.

A. D. Hiebert, F. E. Vermeulen, F. S. Chute, J. (1989). Application of Numerical Modeling to The Simulation of the Electric-Preheat Steam-Drive (EPSD) Process in Athabasca Oil Sands. *J. Canadian Petroleum Technology*, **28**, 74-86.

Killough, J. E., Gonzalez, J. A. (1986). A fully-implicit model for electrically enhanced oil recovery. *SPE 15605 paper presented at the 61st Annual Technical Conference and Exhibition, SPE of AIME,* New Orleans, LA.

McGee, B. C. W., Vermeulen, F. E. (2007). The Mechanisms of Electrical Heating for the Recovery of Bitumen from Oil Sands. *J. Canadian Petroleum Technology*, **46**(1), 28-34.

McGee, B. C. W. (2008). Electro-thermal pilot in the Athabasca oil sands: Theory versus performance. *World Oil* , **229**(11), 47-54.

Pizarro, J. O. S., Trevisan, O. V. (1990). Electrical Heating of Oil Reservoirs: Numerical Simulation and Field Test Results. *J. Petroleum Technology*, **42**(10), 1320-1326.

Todd, J. C., Howell, E. P. (1978). Numerical Simulation of In-situ Electrical Heating to Increase Oil Mobility. *J. Canadian Petroleum Technology,* **17**(2), 31-41.

Torres, J.A., Bogdanov, I.I., Dabir, V. and Kamp, A.M. (2010). Analysis of coupled and fully integrated models for low-frequency electrical heating assisted heavy oil recovery; *paper A019 presented at the European Conference on Mathematics in Oil Recovery, ECMOR XII,* Cambridge, UK, September 2010.

Tran, T. S., Zitha, P.L.J., De Rouffignac, E. (2009). Electromagnetic Assisted Carbonated Water Flooding for Heavy Oil Recovery, *paper 2009-379 presented at the World Heavy Oil Congress*, Puerto La Cruz, Venezuela.

Vinsome, P. K. W., and Shook, G. M. (1993). Multi-Purpose Simulation. *Journal of Petroleum Science and Engineering*, 9(1), 29-38.

Ucok H., Ershaghi, I., Olhoeft, G.R. (1980). Electrical Resistivity of Geothermal Brine. *J. Petroleum Technology,* **32**(4), 717-727.

Yung, Y-Y., Isaacs, E.E., Huang, H., Vandenhoff, D. (2003). Wet electric heating process. *US Patent 6631761.*

APPENDIX A

At a given electrode pattern the electric conductivity field completely defines the distribution of Joule heating power (cf. Eq. A-3) and, hence, drastically influences both the results of preheating and oil production. The complex electric potential field is defined from stationary field equation which reads as:

$$\nabla \cdot (\sigma \cdot \nabla \upsilon) = 0. \quad \text{(A-1)}$$

According to the generalized Archie's law (cf. Archie 1942):

$$\sigma = \sigma_w \varepsilon^m S_w^n f(T), \quad \text{(A-2)}$$

the bulk electric conductivity σ may vary with local water saturation S_w, lithology type according to the formation factor definition $F^{-1}=\alpha\varepsilon^m$, temperature T and also with all factors affecting the water phase (or brine) conductivity σ_w. The latter include for instance the impact of solutes dissolved in connate and injected water. Available experimental data on the temperature dependency demonstrate that one may take as a general rule the nearly linear increase of the conductivity approximately by a factor of 3 for the first 100°C of temperature rise (cf. Ucok et al 1980). Mention also that the colloid properties of clay deposited in separate inclusions and/or layers may affect considerably the bulk reservoir conductivity.

The variation of the water conductivity with the solute concentration depends much on the dissolved mineral. It is worth mentioning that the rate of water conductivity increase with NaCl concentration is nearly constant and its order of magnitude value is 1 S/m per weight percent and more. To provide necessary heating power in case if initial reservoir conductivity is low enough, it may be feasible to modify the conductivity *e.g.* via forced brine circulation.

The resolution of stationary electric charge conservation equation (Eq. A-1) enables to calculate the local heating power source as:

$$J = \sigma |\nabla \upsilon|^2, \quad \text{(A-3)}$$

where J is the resistive power density term. The constitutive pressure-saturation-permeability relations used for current study together with fluid physical properties variation with temperature and pressure can be found in Ref. 3.

APPENDIX B

The electric well index definition (used in TETRAD) is specified below.

Geometrically speaking the electric potential is imposed on the electrode (or the E-well) external radius; regardless the grid configuration (Cartesian or radial) the current term from E-boundary to block centre reads as:

$$I_{iw} = \frac{(V_i - V_w)}{R_i} \quad \text{(B-1)}$$

where R_i is the resistance factor which is defined by:

$$R_i = \frac{T_i}{\sigma_i} \quad \text{(B-2)}$$

$$T_i^{-1} = \frac{2 * \pi * DELW}{\ln\left(\frac{GF * DELD}{r_w}\right)} \quad \text{(B-3)}$$

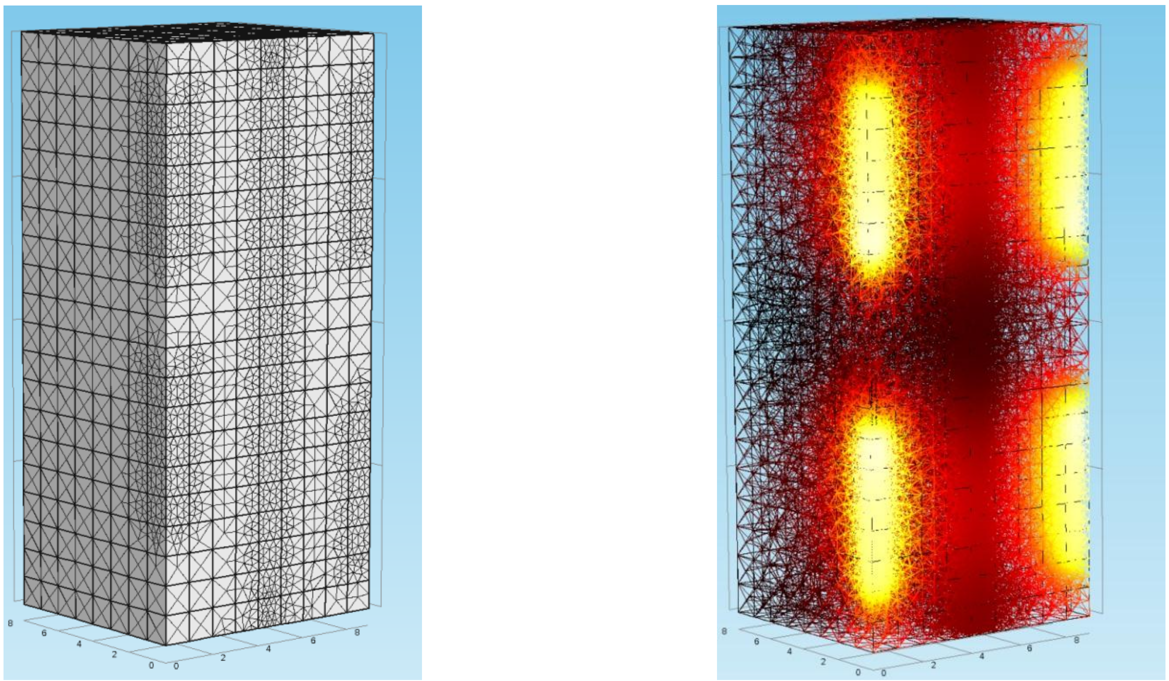
Here DELW is the length in well direction, DELD the block diagonal length in the normal to well direction, r_w the E-well radius, GF the standard Peaceman's geometrical factor. By analogy to the conventional well to reservoir exchange the Eq. B-1 can be written as

$$I_{iw} = EWI_{iw} * (V_i - V_w). \quad \text{(B-4)}$$

If we take a vertical well in the K-direction, a Cartesian grid block i with the sizes ΔX, ΔY and ΔZ in I-, J- and K-direction (as illustrated in Figure 4), then according to the Equations B-1 to B-3 the electrical geometric factor is:

$$EWI_{iw} = \frac{2 * \pi * \sigma_i * \Delta Z}{\ln\left(\frac{GF * \sqrt{\Delta X^2 + \Delta Y^2}}{r_w}\right)} \quad \text{(B-5)}$$

Note that except for the physical property under consideration (here, electric conductivity σ) this is a standard definition for a grid block to well interaction factor (the productivity well index) used in particular in CMG-STARS to compute reservoir to well exchange rate.



(a) **(b)**

Figure 2. Test problem 2: unstructured surface COMSOL grid (a) and wireframe view of the temperature field (b) computed in COMSOL for homogeneous medium properties.

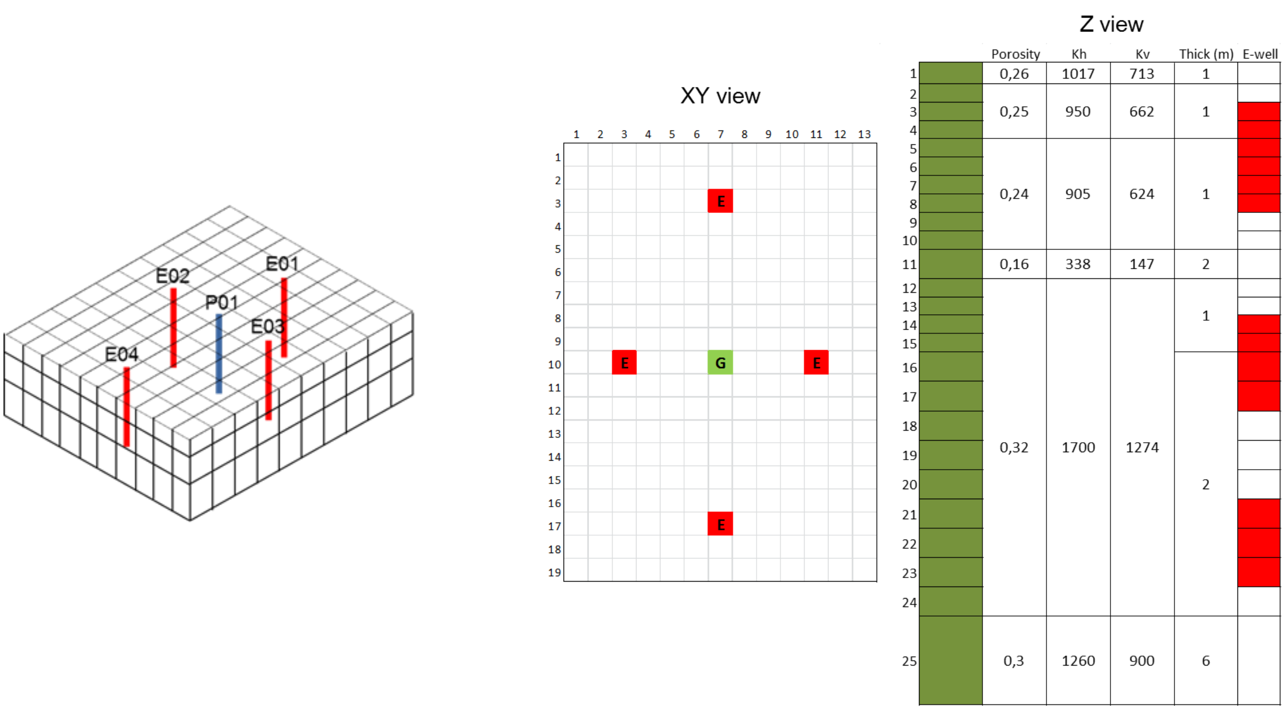
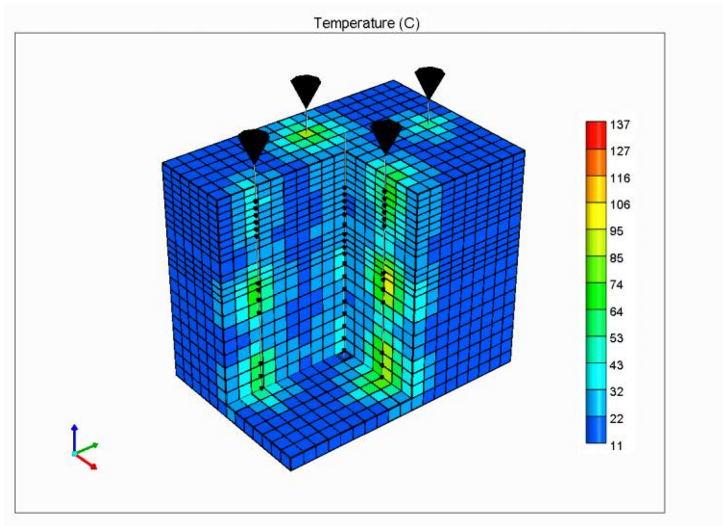


Figure 3. Electrode configuration for problem 3.



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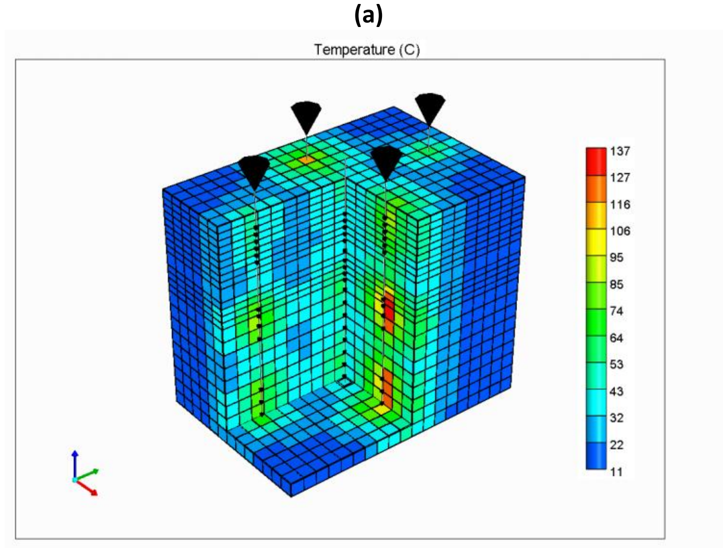

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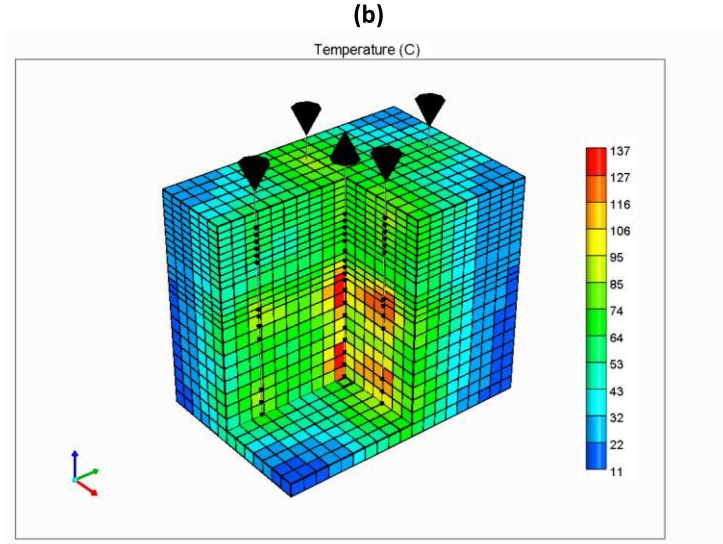

(c)

Figure 4. Test problem 3: temperature field after 4 month (a), 7 month (b) and 12 months of heating and production.