Pore-scale two-phase flow modelling: diffuse interface approach

I.I. Bogdanov, S. Jardel, A.K. Kamp

Open & Experimental Centre for Heavy Oil, University of Pau – UFR Sciences et Techniques – avenue de l'Université – B.P. 1155 – 64013 Pau Cedex, France.

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1 Introduction

Pore-scale modeling of multiphase flow through porous media is addressed most frequently to improve our understanding of flow and transport phenomena in such settings. Besides, it can be used to obtain macro-scale constitutive equations, to provide multiphase flow properties for large scale models, to predict how these properties may vary with rock type, wettability, etc.

The description of a physical interface separating different phases inside a pore volume is a problem of crucial importance for such a modeling. Instead of using a regularization technique to capture the interface, which may affect the results in non-trivial way, the diffuse interface method is based on thermodynamic treatment of phase transition (or phase mixing) zone. As a result, it is a good choice for a numerical technique, handling the morphological changes of the interface [1].

We have used systematically the diffuse interface approach (or equivalently, the field phase model) of two-phase immiscible stable and unstable flow for 2D and 3D computations in a porous medium. We compared numerical solutions to analytical solutions for simple geometries (parallel flat plates and a cylindrical tube) and calculated subsequently phase flows between regular arrays of cylinders or spheres.

In all cases to check a flow regime we calculated phase relative permeabilities. The impact of flow stability, wettability and capillary number on the flow pattern and models computational performance are also presented and discussed.

2 Diffuse interface model

Presented below is only a brief description of Cahn-Hilliard diffuse interface model. For more details of the theory and its formalism see *eg* [1], [2], [3]. The second gradient theory assumes that free energy of a system is a functional of an order parameter φ, its gradient $\nabla\varphi$ and the temperature T. In the case of an isothermal binary fluid, a free energy can be defined for flow configurations where the system is not in equilibrium as a following functional:

$$F(\varphi,\nabla\varphi)=\alpha\int_{\Omega} f\,dV=\alpha\int_{\Omega}\frac{1}{2}\left(|\nabla\varphi|^2+g(\varphi)\right)dV \tag{1}$$

In equation (1) Ω is the region of space occupied by the system and φ is a dimensionless phase-field variable which serves to identify the two fluids with volume fractions $(1+\varphi)/2$ and $(1-\varphi)/2$, a the mixing energy density [N], a following shape for so-called double potential is chosen: $g(\varphi)=(1/4\xi^2)\cdot(\varphi^2-1)^2$. The chemical potential is introduced as:

$$\nu=\delta F/\delta\varphi=\frac{\alpha}{\xi^2}\left[\varphi\left(\varphi^2-1\right)-\xi^2\nabla^2\varphi\right] \tag{2}$$

where ξ is a capillary width [m], that scales with the thickness of the diffuse interface. The fourth order convective Cahn-Hilliard equation describing the evolution of φ reads as:

$$\frac{\partial\varphi}{\partial t}+(\mathbf{u}\cdot\nabla)\,\varphi-\nabla\cdot\left(M(\varphi)\nabla\mu\right)=0 \tag{3}$$

where the mobility $M(\varphi)=M_c(1-\gamma\varphi^2)\,\xi^2$ can be taken without loss of generality at $\gamma=0$. Combining equations (2),(3) and *modified* Navier-Stokes ones for incompressible fluid, one gets finally the model equations to be solved which are:

$$\begin{aligned}&\rho\left(\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\,\mathbf{u}\right)+\nabla p=\nabla\cdot\left[\mu\left(\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right)\right]\\ &\qquad\qquad\qquad\qquad+\nu\cdot\nabla\varphi\\ &\nabla\cdot\mathbf{u}=0\\ &+\text{ equations (2), (3)}\end{aligned} \tag{4}$$

The surface tension is introduced through the integral of the free-energy density across the interface; it relates both above defined model parameters, a and ξ, via equilibrium relation [4]: $\sigma = (2\sqrt{2}/3)\cdot(a/\xi)$.

3 Main results

Promising first results have been obtained for 3D two-phase flow model based on diffuse interface approach (Figure 1). Flow regimes at viscous and capillary limits, for drainage and imbibitions, at favorable and unfavorable to viscous fingering viscosity ratio. Analysis of results expressed as phase relative permeabilities allowed to distinguish classical (Darcy's law based) and "non-classical" flow behavior which generally takes place, at least, locally during stable and unstable displacement.

Additional efforts are needed to extend the application field for 3D diffuse interface model of two-phase pore scale flow. As an example of nearest future work, the locally dominating capillary forces at pore scale can be considered which are responsible for mass exchange by capillary imbibition. Modeling this process in its dynamics may provide better understanding of mass transfer from both qualitative and quantitative viewpoints. Realistic geometry of pores becomes one of key factors for such a modeling.

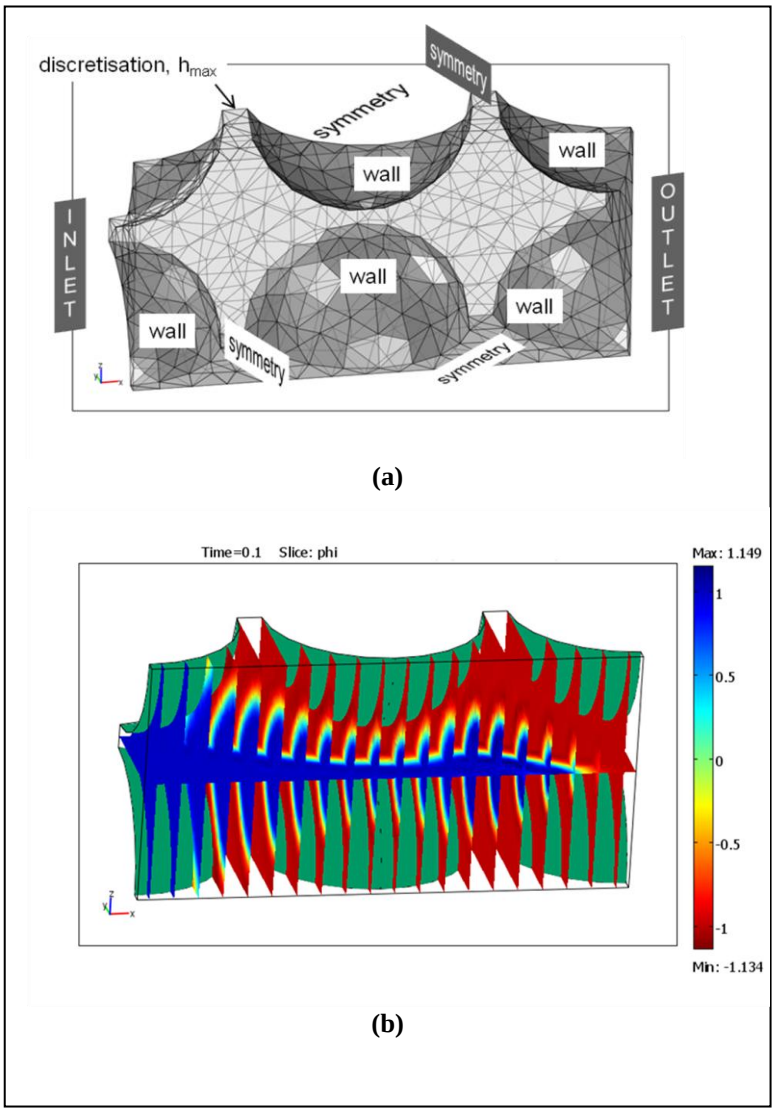

Figure 1. 3D model geometry and boundary conditions (a), viscous limit case phase field at capillary number Ca=0.15 (b).

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