COMSOL 2D Simulation of Heavy Oil Recovery by Steam Assisted Gravity Drainage

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Abstract: Multi-physics flexibility and computational performance of COMSOL gave us the idea to model SAGD ("steam assisted gravity drainage") one of the popular thermal method of oil recovery. The modeling is far from being straightforward and requires solving a system of non-linear PDE for thermal multiphase flow under conditions of thermodynamic (phase) equilibrium. This paper presents main results of our work on constructing the SAGD model and offers the general framework for such kind of modeling. The results of computations in COMSOL are directly juxtaposed with the similar computational results obtained by well-known reservoir simulator STARS.
Keywords: multiphase flow, thermodynamic equilibrium, gravity drainage, oil recovery, PDE application mode.

1. Introduction

Conventional hydrocarbon resources become scarcer every day, while world energy needs continue to increase. For this reason, energy producing companies increasingly exploit non-conventional hydrocarbon resources such as heavy oil and bitumen. Large amounts of such resources exist in for example Canada, Russia and Venezuela. Because of their large viscosity, production of such oils is frequently done through heating, in particular by means of steam injection.

A recovery process that has gained much popularity in recent years is the "steam assisted gravity drainage" (SAGD) [1]. It consists of steam injection through a horizontal well bore into an oil reservoir. The steam penetrates the porous medium surrounding the wellbore, heats the oil which at its turn drains to another horizontal well beneath the injection well. The flow in the porous medium is multiphase (oil, water, gas) and multi-component (one or several hydrocarbon components and water). Although dedicated software for simulation of such processes exists, it lacks the multi-physics flexibility of COMSOL. For this reason we modeled the process using COMSOL. Such modeling is not straightforward, and requires coupling of an energy balance, and phases/components mass balances, the components being under phase equilibrium at reservoir conditions.

Generally speaking, the choice of an application mode for the model equations seems to be not unique in COMSOL. For example, the flow equations can be taken from *Earth Science Module* or modeled via *PDE Application Mode*. Although it would be both interesting and instructive to make a comparative study for various COMSOL functionalities we've preferred for the first time to rely on our base knowledge of reservoir simulation principles and short experience in dealing with COMSOL. Thus the conventional (for reservoir numerical applications) approach underlies the 2D model and in particular, the selection of primary variables and governing equations.

The physical idea of the SAGD, mathematical and numerical models used are presented in the next three sections. The choice of the application mode, the boundary conditions, the finite elements type and the need of solution regularization technique are discussed. Then the first results of 2D SAGD simulation are presented and discussed in the fifth section. In particular, the possible improvements of the model performance are proposed.

2. Physical background

Two physical effects underlie the recovery method. First one is the well-known phenomenon of rapidly decreasing oil viscosity when temperature increases. The second one is gravity based drainage of preheated and hence much more mobile oil to a production well. The thermal energy is provided by steam injection via an injection well. Due to density contrast between steam and oil, the former will flow

upward to the top of reservoir under the action of gravity force while for the same reason preheated oil and liquid water will flow in the opposite direction. Consider the so-called generalized Darcy' law for phase flow through a porous medium, which can be written as

$$u_p = -K\eta_p \cdot \left(\nabla P_p + \rho g e_z\right) \qquad (2.1)$$

where u stands for local phase flow (Darcy' velocity), P is phase pressure, $\eta = k_r / \mu$ relative phase mobility, K and k_r are absolute and phase relative permeability, ρ, μ phase density and viscosity, respectively, g gravity acceleration constant and "*p*" phase index. Hereinafter we'll neglect the difference between the phase pressures which is called capillary pressure (see the discussion on the subject in the subsection 4.1). Initially the reservoir is in vertical (gravity) equilibrium (see for details subsection 3.3 below). After injection of sufficient amount of steam, the "steam chamber" is formed around the injection site, i.e. the region with temperature which corresponds to steam/liquid water equilibrium at local reservoir pressure. Due to relatively high mobility of steam the pressure gradient, ∇P, across the "steam chamber" is small and it is gravity driven flow, $u \propto \eta \cdot \rho g$, which dominates both in the "steam chamber" and in the rest of the reservoir. So to produce the oil one doesn't even need to keep high pressure conditions on the injection well.

3. Mathematical model

The 2D model of three phase two component (water and oil) flow under non-isothermal condition (steam injection) comprises the component mass/total thermal energy conservation equations which are completed by the constitutive relations and typical initial and boundary conditions description.

3.1 Governing equations

Component mass conservation equations are written for the water, which can be presented in liquid (index "*w*") and gas state (index "*g*"), and for the oil which is assumed to be uniform non-volatile liquid (index "*h*"). The equations are as follows

$$\varepsilon \partial_t \left(\rho_w S_w + \rho_g S_g\right) + \nabla \cdot \left(\rho_w u_w + \rho_g u_g\right) = 0 \qquad (3.1)$$

$$\varepsilon \partial_t \left(\rho_h S_h\right) + \nabla \cdot \left(\rho_h u_h\right) = 0 \qquad (3.2)$$

where phase flow u is described by generalized Darcy' law (2.1), ε is porosity, S phase saturation. As the temperature will not be uniform in the reservoir, the total thermal energy conservation equation which includes solid (index "*s*") and fluid phases (index "*f*") contributions under assumption of local thermal equilibrium (one-temperature approach), complements the model

$$\partial_t \left(E_s + E_f\right) + \nabla \cdot \left(U_f - \lambda \nabla T\right) = 0 \qquad (3.3)$$

Here E is volumetric internal energy, U_f is total volumetric flow of thermal energy, λ reservoir thermal conductivity coefficient, T temperature. The total flow U_f comprises fluid phase flows, $U_p = \rho_p h_p$, p=*w,g,h*, where h is specific enthalpy, $h_g = h_w + \Lambda$, Λ is specific heat of water vaporization.

Finally, pore volume conservation constraints phase saturations in usual manner

$$S_w + S_g + S_h = 1 \qquad (3.4)$$

3.2 Constitutive relations

The system of equations (3.1-3.4),(2.1) takes into account the main hydrodynamic features of the SAGD-like process under consideration. It provides the relations between physical variables such as temperature, pressure and volume fractions of fluids in the porous medium (saturations). It has to be completed however with a set of constitutive relations which gives a local phase flows description depending on local pressure, temperature and phase saturations. For relative phase permeabilities the relationships based on *van Genuchten-Mualem* model [3] have been used for water (wetting phase) and gas (non-wetting). Assuming that oil is intermediately wetting phase we've taken advantage of *Brooks-Corey-Burdine* model. *Oliveira* and *Demond* [2] have shown in particular, that this model was one of the best in systematic comparison on available experimental data on three phase relative permeability measurements carried out for last decades.

Conventional data available elsewhere for the physical properties of saturated steam/liquid water system have been used to relate such properties as viscosity, density and enthalpy on pressure and temperature variations. Typical for heavy oil exponentially decaying with

temperature viscosity given by the following relation

$$\mu_h = \mu_0 \exp\{b / T_{,K}\} \qquad (3.5)$$

has been chosen for our purposes. The equation (3.5) is one of the built-in functions in well-known and popular reservoir simulator STARS [4]. Here μ_0 and b are parameters, temperature $T_{,K}$ should be taken in absolute units.

3.3 Initial and boundary conditions

The equations (3.1-3.4),(2.1) have been applied in 2D rectangular region with aspect ratio 2: $(-L \le x \le L,\ 0 \le z \le L)$, and open circular hole in the middle which bear the injection well boundary conditions (see Fig.1). We choose the vertical (gravity) equilibrium state like the initial one. As the capillary pressure is neglected, it implies that only one phase (oil) is mobile at $t=0$ and the pressure undergoes linear variation with depth,

$$P_{ini}(t=0,x,y)=P_{top}+\rho_h g(z_{top}-z) \qquad (3.6)$$

The initial temperature and saturations are uniform

$$T(t=0)=T_{ini}\ ,\ S_w(t=0)=S_m,\ S_h(t=0)=1-S_m \qquad (3.7)$$

where S_m is critical water saturation in a sense that $k_{rw}(S_m)=0$. On the left, right and top boundaries, $(x=\pm L,\ z=L)$, the no-flow and thermal insulation conditions are given. On the bottom, $(z=0)$, the constant (initial) pressure and thermal *convective flow* conditions, $\mathbf{n}\cdot\nabla T=0$, are imposed. Similar to this condition for the flow of thermal energy, no diffusive contribution conditions is given, $\mathbf{n}\cdot\nabla S_p=0$, $p=o,w$, for the phase flows at the bottom. In the subsection 4.3 we discuss the nature of the duffusion-like term in the mass conservation equations (3.1-3.2). The latter boundary condition for phase flow turned out to be a compromise between the nature of corresponding equations and a requirement to apply general thermodynamic equilibrium condition on the bottom boundary.

Finally, on the boundary of circular hole of given radius R_w : $(x-x_w)^2+(z-z_w)^2=R_w^2$, the condition of steam injection at constant pressure, $P_e=P_{ini}+\Delta P$, and corresponding equilibrium temperature, $T_e(P_e)$, taken from the standard tables of saturated steam/water properties, is given. The injected steam quality is equal 1 (no liquid water is injected).

4. Numerical model

In this part of paper the motivation of the application mode choice and key features of COMSOL numerical model are presented and discussed in some details.

4.1 Choice of primary variables and application modes

Our recent experience in the numerical modeling of multiphase flow through porous media and implementation of reservoir simulations in research work turned out to be in favor of certain rules in constructing of the COMSOL model for SAGD-like process. First of all, the pressure-saturation primary variables set has been chosen. Generally in reservoir simulation the pressure variation between injection and production sites is smaller than initial reservoir pressure, $\Delta P << P_{ini}$. Moreover, frequently the capillary pressure, P_c, which can play an important role locally in the flow region, is even smaller, than pressure drop, $P_c << \Delta P$. Under these conditions it is clear that the choice of pressure-pressure primary variables may be prohibitive because of usual computational errors which can affect drastically the numerical solution.

Furthermore, the degenerate case, from the view-point of pressure-pressure variables, of zero capillary pressure is common in practice of reservoir simulations like in particular, in our case.

Then the choice of primary variables implies that *PDE application mode* is currently most appropriate for the model (3.1)- (3.4),(2.1).

4.2 Phase equilibrium model

While the thermal equilibrium between all the phases is overall valid, the saturated steam/liquid water equilibrium which implies that pressure and temperature are not independent, is valid only if both steam and water are presented. In fact, it means that for three phase and two phase sub-regions the set of independent variables is different.

To the best of our knowledge, the variables substitution technique which is used conventionally to fix the problem, is not currently available in COMSOL.

Fortunately, COMSOL offers large enough variety of different means enabling to attack the problem from other sides. There are at least two ways to do this. The first one is to proceed with *near-equilibrium* formulation of the equation (3.1) by splitting it into two equations for gas (steam) and liquid water *phases*, as follows

$$\varepsilon\partial_t\left(\rho_w S_w\right)+\nabla\cdot\left(\rho_w u_w\right)=\mathrm{f}_{\mathrm{cn}} \qquad (4.1)$$

$$\varepsilon\partial_t\left(\rho_g S_g\right)+\nabla\cdot\left(\rho_g u_g\right)=-\mathrm{f}_{\mathrm{cn}} \qquad (4.2)$$

$$\mathrm{f}_{\mathrm{cn}}=\gamma(P_g-P_e) \qquad (4.3)$$

where $P_e = P_e\,(T)$ is saturated steam pressure at given temperature T. Main drawback of this *2 equations approach* is the uncertainty concerning the value of coefficient γ which has to meet some physical and/or mathematical requirements.

The second one is the so-called sequential computation which can be called here *single equation approach*. Summing the equations (3.1) and (3.2) one gets the total flow or in other word, pressure equation. Note, that this equation is *quasi* elliptic and least variable in time with respect to others. The idea of the approach is to recalculate pressure separately and possibly not at each time step and then proceed in usual manner with equations (3.1-3.4),(2.1). Again additional efforts are required to adjust this procedure. Besides that, this approach is only applicable via scripting (see subsection 4.4).

To conclude, our experience shows that the results of both *near-equilibrium* approaches are rather close.

4.3 Artificial diffusion

Remind that we neglect the capillary pressure in our model. Let $\lambda=0$ (equation (3.3)). Then (3.1-3.4),(2.1) becomes the system of hyperbolic equations which admits normally discontinuities in solution.

By default, COMSOL offers to deal with finite elements of 2nd order. It is of no doubt that following this way one arrives easily to the solutions which oscillate and hence demonstrate non-physical behavior. So artificial diffusion is needed to avoid non-monotone solution and again, COMSOL offers the set of mean to avoid non-physical solution. Additional *weak terms* in equations (3.1-3.2) enabled to sufficiently improve the results of computations. Along with this the application of artificial diffusion permitted us to specify appropriate bottom boundary conditions for liquid phase saturations.

4.4 Scripting

Sequential or *single equation approach* (see sunsection 4.2) presumes calculating separately the pressure field in order to better satisfy the near-equilibrium conditions for three phase sub-domain. To do this one needs to use scripting for the adequate problem formulation. Although the real time of computation may increase significantly, the use of script has many various advantages and may enlarge the field of possible applications.

5. Results and Discussion

The main purpose of our work is to reveal the COMSOL applicability to problems of modeling the non-conventional methods of oil recovery. It seems to be not very realistic idea to implement in COMSOL full reservoir simulator capabilities. Our intention was rather to show that there are numerous potential areas of oil recovery where COMSOL can be a useful and successful means of research and design advancement.

SAGD has been taken like an example of possible applications because the methods is based on different strongly coupled physical phenomena and undergoes currently various modifications aimed at its enhancement.

The model has been built progressively. The (two phase) Buckley-Leverett problem was the first successful application. Then the modeling of isothermal gas injection via horizontal well to reservoir saturated by liquid water or oil has been done. Finally, hot water and steam injection has completed our work. Where possible the numerical solutions have been directly juxtaposed with exact solution, mass and energy balance has been verified and the influence of auxiliary numerical model parameters value (like artificial diffusion coefficients or parameter γ in the equation (4.3)) has been tested.

The computations of the SAGD process have been performed on triangular non-structured grid with 1086 vertices and 2088 elements (Fig.1). The main parameters of the model are given in the Table below. The temperature, pressure and gas saturation fields are presented in Figures 2a-2c.

It is the gradual development of a “steam chamber” with nearly uniform temperature and slightly perturbed pressure which makes the method so attractive because the heated oil is displaced from the chamber towards the bottom. Note that there is no need to apply great additional pressure on the injection well (cf

$\Delta P = 0.25$ *bar* and, for example, gravity imposed pressure drop, $\rho_h g L \approx 1.6$ *bar*).

To validate the results of computation in COMSOL for the SAGD process the same computation has been carried out with reservoir simulator STARS on the rectangular structured grid *73*x*1*x*36* with $\Delta x = \Delta z = 0.5$ m and *2628* grid cells. The results of computation in STARS are depicted in Figures 3a-3c.

6. Conclusions

COMSOL model of the SAGD process including three phase non-isothermal flow and water phase transition has been successfully built and tested. Although computational performance of the model is not still competitive at the moment in comparison to the industrial reservoir simulator, the multi-physics flexibility and diversity of COMSOL based applications offer the promising possibilities in research on novel methods of oil recovery.

7. References

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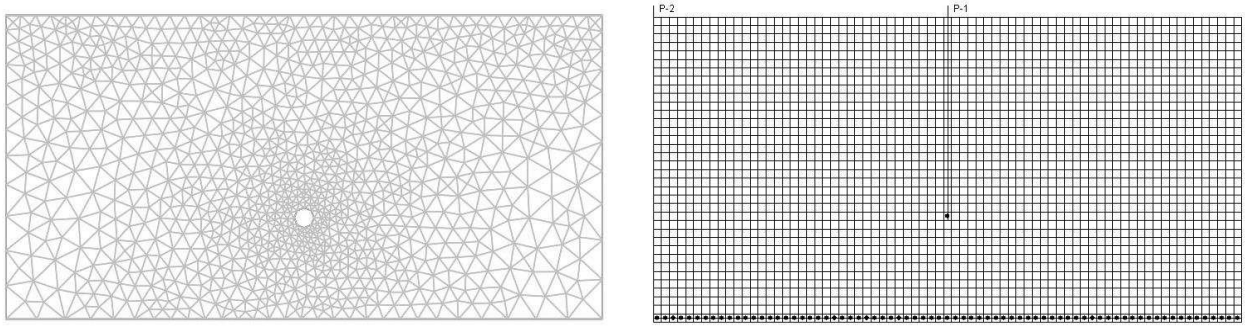
8. Acknowledgements

TOTAL is acknowledged for sponsoring our research activities.

Table 1: Parameters of the SAGD model

Injection temperature, T_e	454	K
Injection pressure, ΔP	0.25	bar
Porosity, ε	0.32	undim.
Absolute permeability, K	1	Darcy
Total thickness, L	18	m
Oil density, ρ_h	900	kg/m^3
Initial oil viscosity	0.1	Pa.s
Reservoir pressure, P_{ini} ($z = z_w$)	10	bar
Initial water saturation, S_m	0.2	undim.
Initial oil saturation, $S_h(t=0)$	0.8	undim.



(a) **(b)**

Figure 1. COMSOL model grid (a) and STARS model grid (b).

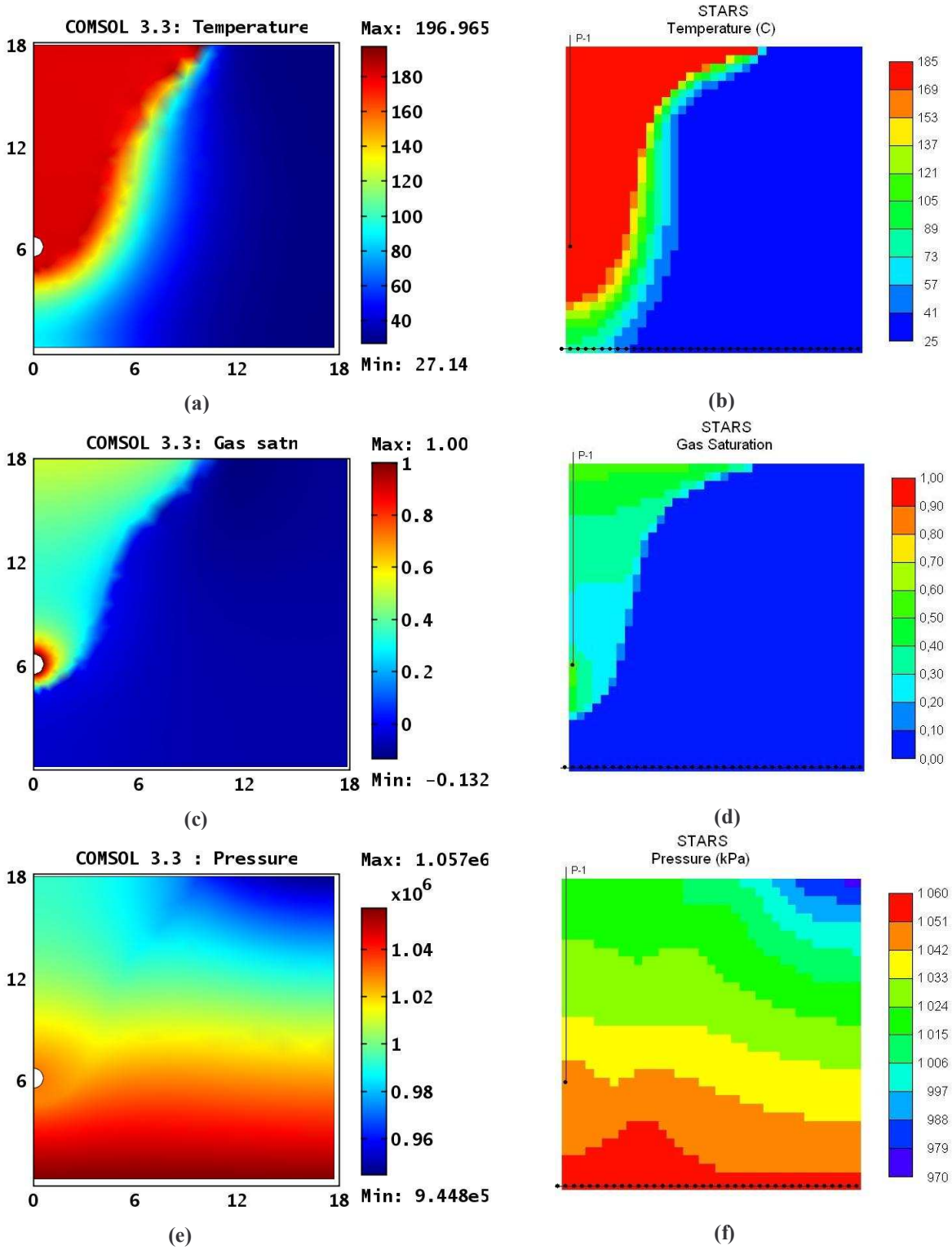


Figure 2. Comparison of COMSOL model to similar STARS model results: temperature fields (a,b), in C, gas saturation fields (c,d) and pressure fields (e,f), in Pa for COMSOL ,(e), and in KPa for STARS, (f), after injection of steam amount equivalent to 4.08 m^3 of liquid water at reservoir conditions